

Two Layered Flagstone Tessellation

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keywords: computational origami, origami tessellation, tiling

Abstract

The origami tessellation is one of origami designs which has polygonal-based tiling pattern. The structure of *the hydrangea tessellation* developed by Shuzo Fujimoto is a well known origami tessellation which has radially expanding polygonal layers[1]. Besides this, a number of variations of origami tessellation have been proposed. *The shrink-rotate algorithm* proposed by Bateman[2] is well known computational tessellation algorithm which constructs crease patterns consist of arrays of simple flat twist-folds from a tiling pattern. Recently, Lang proposed a general construction algorithm for the tessellation in which every visible polygon lies entirely on a single layer and no polygon covers over any visible polygons; it is called *flagstone tessellation*[3].

There is a class of origami tessellation which look like some polygons are lifted up as shown in Fig. 1. In contrast with flagstone tessellation, each polygonal area is classified into upper or lower layer. The areas classified upper layer are surrounded by mountain folds and look slightly lifted up by cornerstone like small polygons. We name these tessellations *two layered flagstone tessellation*.

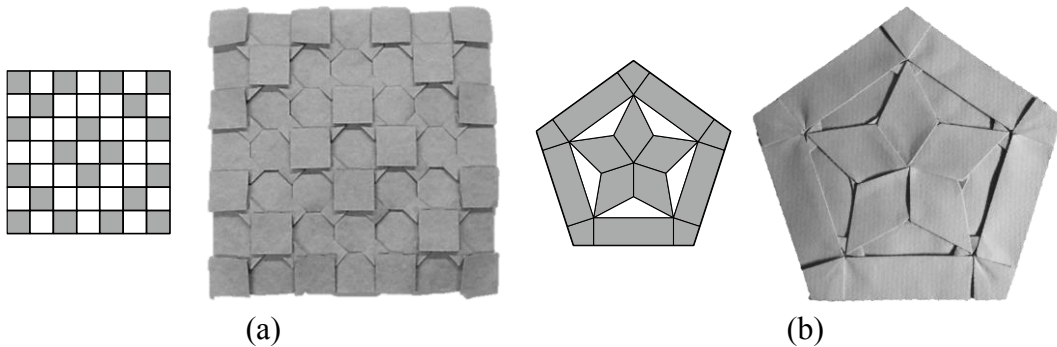


Figure 1 Examples of two layered flagstone tessellation. (a) Grid pattern. (b) A star surrounded by a pentagonal frame

In this paper, we propose an algorithm consists of following steps (Fig. 2) for this tessellation.

- Step 1 Input a tiling pattern with black (upper layer) and white (lower layer) polygonal tiles.
- Step 2 Shrink each tile about its corresponding vertex of the reciprocal figure.
- Step 3 Insert pleating pattern between edges of the shrunk tiles formerly touching each other.
- Step 4 Insert crease pattern modules which fold the corner structure (dark gray region in Fig. 2(3)) to make upper layer lifted up and connects the pleats inserted at step 3.

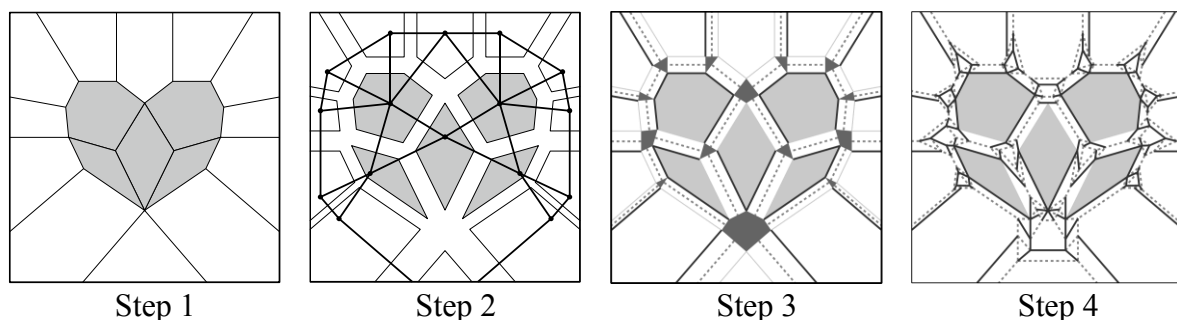


Figure 2 Flow of construction algorithm for two layered flagstone tessellation. Solid lines and dotted lines correspond to mountain and valley folds respectively.

At step 1, the polygonal tiling pattern has to satisfy the spider web condition [4] as with flagstone tessellation. Inserted pleats at step 2 are single pleat created with a pair of mountain and valley folds, whereas flagstone tessellation inserts double pleat. At step 3, if a black tile formerly touches a white tile via an edge, mountain parity is assigned to the black tile's edge. Otherwise, mountain parity is able to be assigned to either edge. At step 4, crease pattern of pleats around a vertex are connected in a module which is placed in area enclosed by the vertices of shrunk polygons. In the module, there exists a polygon serving as a cornerstone, i.e. a part of the module is folded into a cornerstone like shape which lifts up upper layer polygons (Fig. 3). In this paper, we do not consider failure cases where the folding lines intersect in the modules. This issue is left as an open problem.

With above mentioned algorithm, we can make two layered flagstone tessellation from a figure drawn with two colors if the regions can be split into polygonal tiles which satisfy the spider web condition. The example of a star surrounded by pentagonal frame in Fig. 1(b) is split into twenty tiles which satisfy the condition.

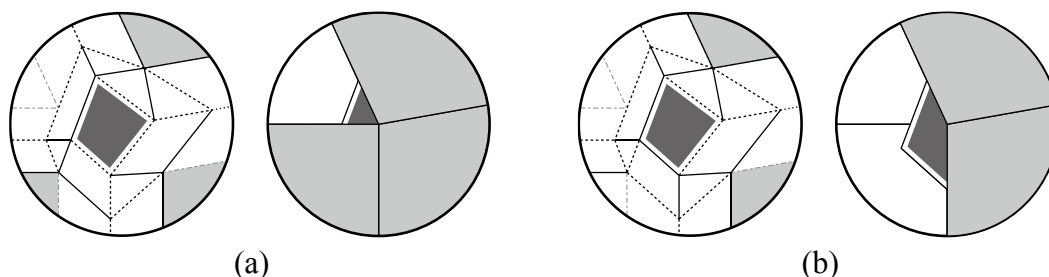


Figure 3 Two examples of module crease patterns and their folded states. White regions surrounded by light-gray edges are lower polygons. Light-gray regions are upper polygons, and dark-gray regions are cornerstone polygons.

Reference

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